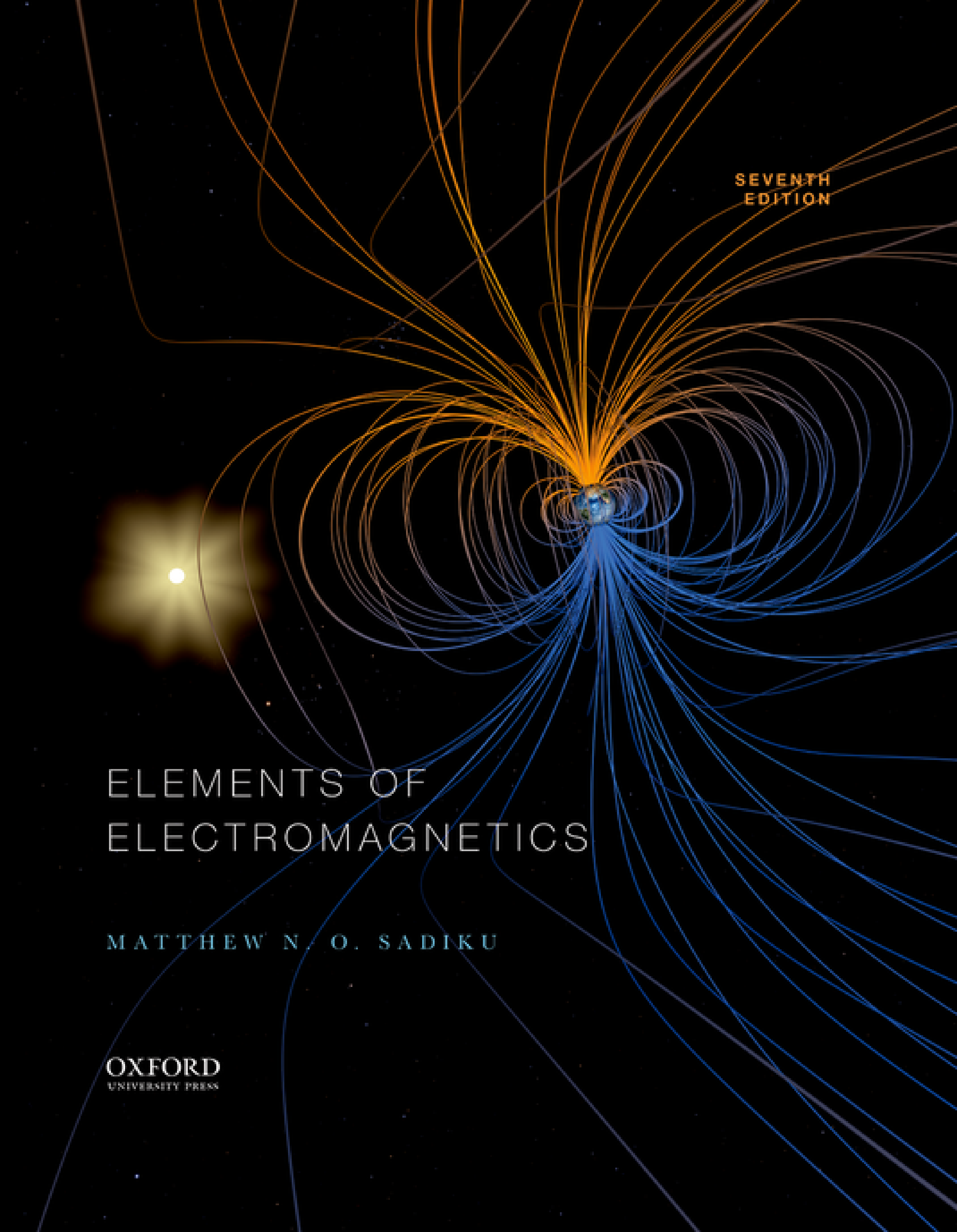

SEVENTH EDITION

ELEMENTS OF ELECTROMAGNETICS

MATTHEW N. O. SADIKU

OXFORD UNIVERSITY PRESS

PRACTICAL APPLICATIONS

Some of the real-life applications covered in this book are listed in order of appearance.

- Applications of electrostatics (Section 4.1)
- Electrostatic separation of solids (Example 4.3)
- Electrostatic discharge (ESD) (Section 4.11)
- Electrostatic shielding (Section 5.9B)
- High dielectric constant materials (Section 5.10)
- Graphene (Section 5.11) ***NEW***
- Electrohydrodynamic pump (Example 6.1)
- Xerographic copying machine (Example 6.2)
- Parallel-plate capacitor, coaxial capacitor, and spherical capacitor (Section 6.5)
- RF MEMS (Section 6.8) (Chapter 12 opener) ***NEW***
- Ink-jet printer (Problem 6.52)
- Microstrip lines (Sections 6.7, 11.8, and 14.6)
- Applications of magnetostatics (Section 7.1)
- Coaxial transmission line (Section 7.4C)
- Lightning (Section 7.9)
- Polywells (Section 7.10) ***NEW***
- Magnetic resonant imaging (MRI) (Chapter 8 opener)
- Magnetic focusing of a beam of electrons (Example 8.2, Figure 8.2)
- Velocity filter for charged particles (Example 8.3, Figure 8.3)
- Inductance of common elements (Table 8.3)
- Electromagnet (Example 8.16)
- Magnetic levitation (Section 8.12)
- Hall effect (Section 8.13) ***NEW***
- Direct current machine (Section 9.3B)
- Memristor (Section 9.8) ***NEW***
- Optical nanocircuits (Section 9.9) ***NEW***
- Homopolar generator disk (Problem 9.14)
- Microwaves (Section 10.11)
- Radar (Sections 10.11 and 13.9)
- 60 GHz technology (Section 10.12) ***NEW***
- Bioelectromagnetics (Chapter 11 opener)
- Coaxial, two-line, and planar lines (Figure 11.1, Section 11.2)
- Quarter-wave transformer (Section 11.6A)
- Data cables (Section 11.8B)
- Metamaterials (Section 11.9) ***NEW***
- Microwave imaging (Section 11.10) ***NEW***
- Optical fiber (Section 12.9)
- Cloaking and invisibility (Section 12.10) ***NEW***
- Smart antenna (Chapter 13 opener)
- Typical antennas (Section 13.1, Figure 13.2)
- Electromagnetic interference and compatibility (Section 13.10)
- Grounding and filtering (Section 13.10)

- Textile antennas and sensors (Section 13.11) ***NEW***
- RFID (Section 13.12) ***NEW***
- Commercial EM software—FEKO (Section 14.7) ***NEW***
- COMSOL Multiphysics (Section 14.8) ***NEW***
- CST Microwave Studio (Section 14.9) ***NEW***

PHYSICAL CONSTANTS

Quantity (Units)	Symbol	Best Experimental Value*	Approximate Value for Problem Work
Permittivity of free space (F/m)	ε_o	8.854×10^{-12}	$\frac{10^{-9}}{36\pi}$
Permeability of free space (H/m)	μ_o	$4\pi \times 10^{-7}$	12.6×10^{-7}
Intrinsic impedance of free space (Ω)	η_o	376.6	120π
Speed of light in vacuum (m/s)	c	2.998×10^{8}	3×10^{8}
Electron charge (C)	e	-1.6022×10^{-19}	-1.6×10^{-19}
Electron mass (kg)	m_e	9.1093×10^{-31}	9.1×10^{-31}
Proton mass (kg)	m_p	1.6726×10^{-27}	1.67×10^{-27}
Neutron mass (kg)	m_n	1.6749×10^{-27}	1.67×10^{-27}
Boltzmann constant (J/K)	κ	1.38065×10^{-23}	1.38×10^{-23}
Avogadro number (/kg-mole)	N	6.0221×10^{23}	6×10^{23}
Planck constant (J · s)	h	6.626×10^{-34}	6.62×10^{-34}
Acceleration due to gravity (m/s^2)	g	9.80665	9.8
Universal constant of gravitation N (m/kg)^2	G	6.673×10^{-11}	6.66×10^{-11}
Electron-volt (J)	eV	1.602176×10^{-19}	1.6×10^{-19}

* Values recommended by CODATA (Committee on Data for Science and Technology, Paris).

ELEMENTS OF ELECTROMAGNETICS

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ELEMENTS OF ELECTROMAGNETICS

SEVENTH EDITION

MATTHEW N. O. SADIKU
Prairie View A&M University

New York • Oxford
OXFORD UNIVERSITY PRESS

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To my wife, Kikelomo

BRIEF TABLE OF CONTENTS

CONTENTS

[†] Indicates sections that may be skipped, explained briefly, or assigned as homework if the text is covered in one semester.

PART 4: WAVES AND APPLICATIONS

PREFACE

This new edition is intended to provide an introduction to engineering electromagnetics (EM) at the junior or senior level. Although the new edition improves on the previous editions, the core of the subject of EM has not changed. The fundamental objective of the first edition has been retained: to present EM concepts in a clearer and more interesting manner than other texts. This objective is achieved in the following ways:

1. To avoid complicating matters by covering EM and mathematical concepts simultaneously, vector analysis is covered at the beginning of the text and applied gradually. This approach avoids breaking in repeatedly with more background on vector analysis, thereby creating discontinuity in the flow of thought. It also separates mathematical theorems from physical concepts and makes it easier for the student to grasp the generality of those theorems. Vector analysis is the backbone of the mathematical formulation of EM problems.

2. Each chapter opens either with a historical profile of some electromagnetic pioneers or with a discussion of a modern topic related to the chapter. The chapter starts with a brief introduction that serves as a guide to the whole chapter and also links the chapter to the rest of the book. The introduction helps the students see the need for the chapter and how it relates to the previous chapter. Key points are emphasized to draw the reader's attention. A brief summary of the major concepts is discussed toward the end of the chapter.

3. To ensure that students clearly get the gist of the matter, key terms are defined and highlighted. Important formulas are boxed to help students identify essential formulas.

4. Each chapter includes a reasonable amount of solved examples. Since the examples are part of the text, they are clearly explained without asking the reader to fill in missing steps. In writing out the solution, we aim for clarity rather than efficiency. Thoroughly worked out examples give students confidence to solve problems themselves and to learn to apply concepts, which is an integral part of engineering education. Each illustrative example is followed by a problem in the form of a Practice Exercise, with the answer provided.

5. At the end of each chapter are ten review questions in the form of multiple-choice objective items. Open-ended questions, although they are intended to be thought-provoking, are ignored by most students. Objective review questions with answers immediately following them provide encouragement for students to do the problems and gain immediate feedback. A large number of problems are provided and are presented in the same order as the material in the main text. Approximately 20 to 25 percent of the problems in this edition have been replaced. Problems of intermediate difficulty are identified by a single asterisk; the most difficult problems are marked with a double asterisk. Enough problems are provided to allow the instructor to choose some as examples and assign some as homework problems. Answers to odd-numbered problems are provided in Appendix E.

6. Since most practical applications involve time-varying fields, six chapters are devoted to such fields. However, static fields are given proper emphasis because they are special cases of dynamic fields. Ignorance of electrostatics is no longer acceptable because there are large industries, such as copier and computer peripheral manufacturing, that rely on a clear understanding of electrostatics.

7. The last section in each chapter is devoted to applications of the concepts covered in the chapter. This helps students see how concepts apply to real-life situations.

8. The last chapter covers numerical methods with practical applications and MATLAB programs. This chapter is of paramount importance because most practical problems are only solvable using numerical techniques. Since MATLAB is used throughout the book, an introduction to MATLAB is provided in Appendix C.

9. Over 130 illustrative examples and 300 figures are given in the text. Some additional learning aids such as basic mathematical formulas and identities are included in Appendix A. Another guide is a special note to students, which follows this preface.

NEW TO THE SIXTH EDITION

- Five new Application Notes designed to explain the real-world connections between the concepts discussed in the text.
- A revised Math Assessment test, for instructors to gauge their students' mathematical knowledge and preparedness for the course.
- New and updated end-of-chapter problems.

Solutions to the end-of-chapter problems and the Math Assessment, as well as PowerPoint slides of all figures in the text, can be found at the Oxford University Press Ancillary Resource Center.

Students and professors can view Application Notes from previous editions of the text on the book's companion website www.oup.com/us/sadiku.

Although this book is intended to be self-explanatory and useful for self-instruction, the personal contact that is always needed in teaching is not forgotten. The actual choice of course topics, as well as emphasis, depends on the preference of the individual instructor. For example, an instructor who feels that too much space is devoted to vector analysis or static fields may skip some of the materials; however, the students may use them as reference. Also, having covered Chapters 1 to 3, it is possible to explore Chapters 9 to 14. Instructors who disagree with the vector-calculus-first approach may proceed with Chapters 1 and 2, then skip to Chapter 4, and refer to Chapter 3 as needed. Enough material is covered for two-semester courses. If the text is to be covered in one semester, covering Chapters 1 to 9 is recommended; some sections may be skipped, explained briefly, or assigned as homework. Sections marked with the dagger sign (†) may be in this category.

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I owe special thanks for those professors and students who have used earlier editions of the book. Please keep sending those errors directly to the publisher or to me at sadiku@ieee.org.

—Matthew N.O. Sadiku
Prairie View, Texas

A NOTE TO THE STUDENT

Electromagnetic theory is generally regarded by students as one of the most difficult courses in physics or the electrical engineering curriculum. But this conception may be proved wrong if you take some precautions. From experience, the following ideas are provided to help you perform to the best of your ability with the aid of this textbook:

1. Pay particular attention to Part 1 on vector analysis, the mathematical tool for this course. Without a clear understanding of this section, you may have problems with the rest of the book.

2. Do not attempt to memorize too many formulas. Memorize only the basic ones, which are usually boxed, and try to derive others from these. Try to understand how formulas are related. There is nothing like a general formula for solving all problems. Each formula has limitations owing to the assumptions made in obtaining it. Be aware of those assumptions and use the formula accordingly.

3. Try to identify the key words or terms in a given definition or law. Knowing the meaning of these key words is essential for proper application of the definition or law.

4. Attempt to solve as many problems as you can. Practice is the best way to gain skill. The best way to understand the formulas and assimilate the material is by solving problems. It is recommended that you solve at least the problems in the Practice Exercise immediately following each illustrative example. Sketch a diagram illustrating the problem before attempting to solve it mathematically. Sketching the diagram not only makes the problem easier to solve, but also helps you understand the problem by simplifying and organizing your thinking process. Note that unless otherwise stated, all distances are in meters. For example $(2, -1, 5)$ actually means $(2 \text{ m}, -1 \text{ m}, 5 \text{ m})$.

You may use MATLAB to do number crunching and plotting. A brief introduction to MATLAB is provided in Appendix C.

A list of the powers of 10 and Greek letters commonly used throughout this text is provided in the tables located on the inside cover. Important formulas in calculus, vectors, and complex analysis are provided in Appendix A. Answers to odd-numbered problems are in Appendix E.

ABOUT THE AUTHOR

Matthew N. O. Sadiku received his BSc degree in 1978 from Ahmadu Bello University, Zaria, Nigeria, and his MSc and PhD degrees from Tennessee Technological University, Cookeville, Tennessee, in 1982 and 1984, respectively. From 1984 to 1988, he was an assistant professor at Florida Atlantic University, Boca Raton, Florida, where he did graduate work in computer science. From 1988 to 2000, he was at Temple University, Philadelphia, Pennsylvania, where he became a full professor. From 2000 to 2002, he was with Lucent/Avaya, Holmdel, New Jersey, as a system engineer and with Boeing Satellite Systems, Los Angeles, California, as a senior scientist. He is currently a professor of electrical and computer engineering at Prairie View A&M University, Prairie View, Texas.

He is the author of over 370 professional papers and over 70 books, including *Elements of Electromagnetics* (Oxford University Press, 7th ed., 2018), *Fundamentals of Electric Circuits* (McGraw-Hill, 6th ed., 2017, with C. Alexander), *Computational Electromagnetics with MATLAB* (CRC, 4th ed., 2018), *Metropolitan Area Networks* (CRC Press, 1995), and *Principles of Modern Communication Systems* (Cambridge University Press, 2017, with S. O. Agbo). In addition to the engineering books, he has written Christian books including *Secrets of Successful Marriages, How to Discover God's Will for Your Life*, and commentaries on all the books of the New Testament Bible. Some of his books have been translated into French, Korean, Chinese (and Chinese Long Form in Taiwan), Italian, Portuguese, and Spanish.

He was the recipient of the 2000 McGraw-Hill/Jacob Millman Award for outstanding contributions in the field of electrical engineering. He was also the recipient of Regents Professor award for 2012–2013 by the Texas A&M University System. He is a registered professional engineer and a fellow of the Institute of Electrical and Electronics Engineers (IEEE) "for contributions to computational electromagnetics and engineering education." He was the IEEE Region 2 Student Activities Committee Chairman. He was an associate editor for IEEE Transactions on Education. He is also a member of the Association for Computing Machinery (ACM) and the American Society of Engineering Education (ASEE). His current research interests are in the areas of computational electromagnetics, computer networks, and engineering education. His works can be found in his autobiography, *My Life and Work* (Trafford Publishing, 2017) or on his website, www.matthewsadiku.com. He currently resides with his wife Kikelomo in Hockley, Texas. He can be reached via email at sadiku@ieee.org.

MATH ASSESSMENT

1.1 Let θ be the angle between the vectors **A** and **B**. What can be said about θ if (i) $|\mathbf{A} + \mathbf{B}| < |\mathbf{A} - \mathbf{B}|$, (ii) $|\mathbf{A} + \mathbf{B}| = |\mathbf{A} - \mathbf{B}|$, (iii) $|\mathbf{A} + \mathbf{B}| > |\mathbf{A} - \mathbf{B}|$?

1.2 Two sides of a parallelogram *ABCD* denoted as $\mathbf{p} = 5\boldsymbol{a}_x$ and $\mathbf{q} = 3\boldsymbol{a}_x + 4\boldsymbol{a}_y$ are shown in Figure MA-1 Let the diagonals intersect at O and make an angle α. Find the coordinates of O and the magnitude of α. Based on the value of α, what can we call *ABCD*?

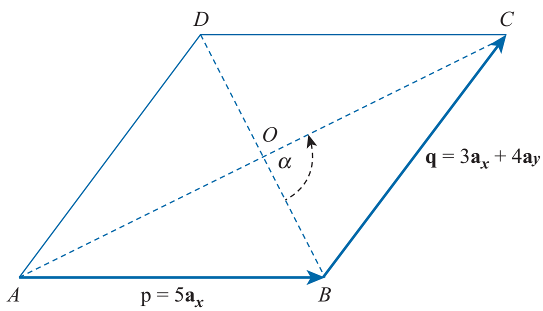

FIGURE MA-1 Parallelogram *ABCD*.

1.3 What is the distance R between the two points $A(3, 5, 1)$ and $B(5, 7, 2)$? Also find its reciprocal, $\frac{1}{R}$.

1.4 What is the distance vector $\mathbf{R}_{AB}$ from $A(3, 7, 1)$ to $B(8, 19, 2)$ and a unit vector $\mathbf{a}_{AB}$ in the direction of $\mathbf{R}_{AB}$?

1.5 Find the interval of values x takes so that a unit vector $\mathbf{u}$ satisfies $|(x - 2)\mathbf{u}| < |3\mathbf{u}|$.

1.6 There are four charges in space at four points A, B, C, and D, each 1 m from *every* other. You are asked to make a selection of coordinates for these charges. How do you place them in space and select their coordinates? There is no unique way.

1.7 A man driving a car starts at point O, drives in the following pattern

15 km northeast to point A,
20 km southwest to point B,
25 km north to C,
10 km southeast to D,
15 km west to E, and stops.

How far is he from his starting point, and in what direction?

1.8 A unit vector $\mathbf{a}_n$ makes angles α, β, and γ with the x-, y-, and z-axes, respectively. Express $\mathbf{a}_n$ in the rectangular coordinate system. Also express a nonunit vector $\overrightarrow{OP}$ of length ℓ parallel to $\mathbf{a}_n$.

1.9 Three vectors $\mathbf{p}$, $\mathbf{q}$, and $\mathbf{r}$ sum to a zero vector and have the magnitude of 10, 11, and 15, respectively. Determine the value of $\mathbf{p}\cdot\mathbf{q}+\mathbf{q}\cdot\mathbf{r}+\mathbf{r}\cdot\mathbf{p}$.

1.10 An experiment revealed that the point $Q(x', y', z')$ is 4 m from $P(2, 1, 4)$ and that the vector $\overrightarrow{QP}$ makes 45.5225°, 59.4003°, and 60° with the x-, y-, and z-axes, respectively. Determine the location of Q.

1.11 In a certain frame of reference with x-, y-, and z-axes, imagine the first octant to be a room with a door. Suppose that the height of the door is h and its width is ρ. The top-right corner P of the door when it is shut has the rectangular coordinates $(\rho, 0, h)$. Now if the door is turned by angle ϕ, so we can enter the room, what are the coordinates of P? What is the length of its diagonal $r = \overline{OP}$ in terms of ρ and z? Suppose the vector $\overrightarrow{OP}$ makes an angle θ with the z-axis; express ρ and h in terms of r and θ.

1.12 Consider two vectors $\mathbf{p} = \overrightarrow{OP}$ and $\mathbf{q}=\overrightarrow{OQ}$ in Figure MA-2. Express the vector $\overrightarrow{GR}$ in terms of $\mathbf{p}$ and $\mathbf{q}$. Assume that $\angle ORQ = 90°$.

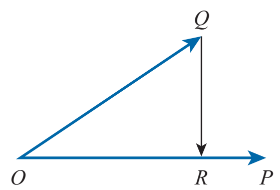

FIGURE MA-2 Orthogonal projection of one vector over another.

1.13 Consider the equations of two planes:

$$3x - 2y - z = 8$$
$$2x + y + 4z = 3$$

Let them intersect along the straight line ℓ. Obtain the coordinates of the points where ℓ meets the xy– and the yz–planes. Also determine the angle between ℓ and the xz-plane.

1.14 Given two vectors $\mathbf{p} = \mathbf{a}_x + \mathbf{a}_y$ and $\mathbf{q} = \mathbf{a}_y + \mathbf{a}_z$ of equal length, find a third vector $\mathbf{r}$ such that it has the same length and the angle between any two of them is 60°.

1.15 Given $\mathbf{A} = 2xy\,\mathbf{a}_x + 3zy\,\mathbf{a}_y + 5z\,\mathbf{a}_z$ and $\mathbf{B} = \sin x\,\mathbf{a}_x + 2y\,\mathbf{a}_y + 5y\,\mathbf{a}_z$, find (i) $\nabla\cdot\mathbf{A}$, (ii) $\nabla\times\mathbf{A}$, (iii) $\nabla\cdot\nabla\times\mathbf{A}$, and (iv) $\nabla\cdot(\mathbf{A}\times\mathbf{B})$.

2.1 A triangular plate of base $b = 5$ and height $h = 4$ shown in Figure MA-3 is uniformly charged with a uniform surface charge density $\rho_s = 10$ C/m^2. You are to cut a rectangular piece so that maximum amount of charge is taken out. What should be the dimensions x and y of the rectangle? What is the magnitude of the charge extracted out?

2.2 Consider two fixed points $F_1(-c, 0)$ and $F_2(c, 0)$ in the xy-plane. Show that the locus of a point $P(x, y)$ that satisfies the constraint that the sum $PF_1 + PF_2$ remains constant and is equal to $2a$ is an ellipse. The equipotential loci due to a uniform line charge of length $2c$ are family of ellipses in the plane containing the charge. This problem helps in proving it.

2.3 Show that the ordinary angle subtended by a closed curve lying in a plane at a point P is 2π radians if P is enclosed by the curve and zero if not.

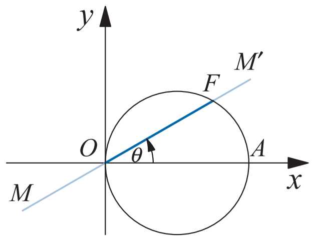

FIGURE MA-3 A rectangular piece cut out from a triangular plate.

2.4 Show that the solid angle subtended by a closed surface at a point P is 4π steradians if P is enclosed by the closed surface and zero if not.

2.5 The electrostatic potential $V(r)$ is known to obey the equation $V(r) = 2V(2r)$ with the boundary condition $V(5) = 3$ volts. Determine $V(15)$.

2.6 Evaluate the indefinite integrals (i) $\int \operatorname{cosec}\theta \, d\theta$ and (ii) $\int \sec\theta \, d\theta$. Ignore the arbitrary constant.

2.7 A liquid drop is in the form of an ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$ shown in Figure MA-4 and is filled with a charge of nonuniform density $\rho_v = x^2$ C/m^3. Find the total charge in the drop.

2.8 Two families of curves are said to be orthogonal to each other if they intersect at 90°. Given a family $y^2 = cx^3$, find the equation for orthogonal trajectories and plot three to four members of each on the same graph.

2.9 Consider a vector given by $\mathbf{E} = (4xy + z)\mathbf{a}_x + 2x^2\mathbf{a}_y + x\mathbf{a}_z$. Find the line integral from $A(3, 7, 1)$ to $B(8, 9, 2)$ by (i) evaluating the line integral $V_{AB} = -\int_A^B \mathbf{E} \cdot \mathbf{dl}$ along the line joining A to B and (ii) evaluating $\left\{ -\int_A^C \mathbf{E} \cdot \mathbf{dl} - \int_C^D \mathbf{E} \cdot \mathbf{dl} - \int_D^B \mathbf{E} \cdot \mathbf{dl} \right\}$, where the stopovers C and D are $C(8, 7, 1)$ and $D(8, 9, 1)$.

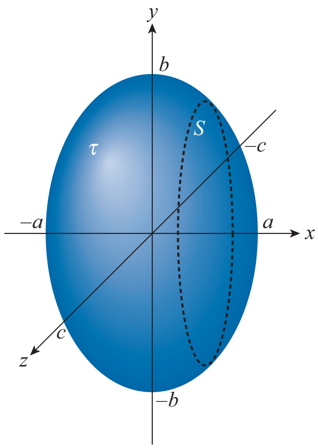

FIGURE MA-4 A non uniformly charged liquid drop.

2.10 Find the trigonometric Fourier series of a function $f(x) = x + x^2$ defined over the interval $-\pi < x < \pi$.

2.11 In a certain electrostatic system, there are found an infinite set of image point charges. The field intensity at a point may be written as

$$E = A \sum_{m=1}^{\infty} \sum_{n=m}^{\infty} \frac{(-1)^{(n-1)}}{n^2}$$

Simplify the double summation.
Hint: Integrating the following series term by term and substituting $x = 1$ helps in finding the result.

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots + - \cdots$$

2.12 Solve the differential equation

$$\frac{d^2V(x)}{dx^2} = \frac{k}{\sqrt{V(x)}}$$

subject to the boundary conditions $\left.\frac{dV}{dx}\right|_{x=0} = 0$ and $V(0) = 0$. Assume that k is a constant.

3.1 The location of a moving charge is given by the time-varying radius vector $\mathbf{r} = 2 \cos t\, \mathbf{a}_x + 2 \sin t\, \mathbf{a}_y + 3t\mathbf{a}_z$. Describe the trajectory of motion. Find the velocity and acceleration vectors at any instant t. In particular, indicate their directions at the specific instants $t = 0$ and $t = \pi/2$. Find their magnitudes at any instant.

3.2 The magnetic field strength $H(z)$ at a point on the z-axis shown in Figure MA-5 is proportional to the sum of cosine of angles and is given by $H = k(\cos\theta_1 + \cos\theta_2)$. Find $H(0)$. Also show that if $a \ll \ell$, $H(\pm\ell) \approx \frac{1}{2}H(0)$. This helps in finding the magnetic field along the axis of a long solenoid.

3.3 Suppose it is suggested that $\mathbf{B} = r(\mathbf{k} \times \mathbf{r})$ is the magnetic flux density vector, where $\mathbf{k}$ is a constant vector and $\mathbf{r} = r\mathbf{a}_r$ verify if it is solenoidal.

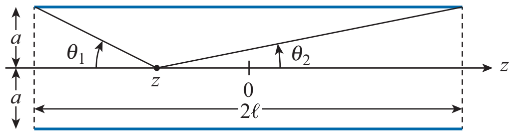

FIGURE MA-5 Toward finding magnetic field along the axis of a solenoid.

3.4 Evaluate the line integral $\oint_C \frac{(x+y)dx + (x-y)dy}{x^2+y^2}$ where C is the circle $x^2 + y^2 = a^2$ of constant radius a.

3.5 Evaluate the line integral $\oint_C \frac{xdy - ydx}{x^2+y^2}$ where C is a closed curve (i) encircling the origin n times, (ii) not enclosing the origin.

4.1 Show that $\nabla \cdot \nabla \times \mathbf{A} = 0$.

4.2 Show that $\nabla \times \nabla\psi = \mathbf{0}$.

4.3 Given that the imaginary unit is $j = \sqrt{-1}$ and that $x = j^j$, could the value of x be real? If so, is it unique? Can x have one value in the interval (100, 120)?

4.4 Show that $\nabla \cdot \mathbf{A} \times \mathbf{B} = \mathbf{B} \cdot \nabla \times \mathbf{A} - \mathbf{A} \cdot \nabla \times \mathbf{B}$.

4.5 Use *De Moivre's theorem* to prove that $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$:

4.6 Determine $\sqrt{j}$.

4.7 Determine $\sqrt{j}$ using the Euler formula.

4.8 Find the phasors for the following field quantities:

(a) $E_x(z, t) = E_o \cos(\omega t - \beta z + \phi)$ (V/m)

(b) $E_y(z, t) = 100e^{-3z}\cos(\omega t - 5z + \pi/4)$ (V/m)

(c) $H_x(z, t) = H_o \cos(\omega t + \beta z)$ (A/m)

(d) $H_y(z, t) = 120\pi e^{-5z}\cos(\omega t + \beta z + \phi_h)$ (A/m)

4.9 Find the instantaneous time domain sinusoidal functions corresponding to the following phasors:

(a) $E_x(z) = E_o e^{j\beta z}$ (V/m)

(b) $E_y(z) = 100e^{-3z}e^{-j5z}$ (V/m)

(c) $I_s(z) = 5 + j4$ (A)

(d) $V_s(z) = j10e^{j\pi/3}$ (V)

4.10 Write the phasor expression $\tilde{I}$ for the following current using a cosine reference.

(a) $i(t) = I_o \cos(\omega t - \pi/6)$

(b) $i(t) = I_o \sin(\omega t + \pi/3)$

4.11 In a certain resonant cavity, the resonant modes are described by a triplet of nonnegative integers m, n, and p. Find possible solutions under the inequality constraints,

$$mn + np + pm \neq 0$$

$$\frac{13}{16} \leq \frac{m^2}{4} + \frac{n^2}{9} + p^2 \leq \frac{5}{4}$$

4.12 A voltage source $V(t) = 100 \cos(6\pi 10^9 t - 45°)$ (V) is connected to a series RLC circuit, as shown in Figure MA-6. Given $R = 10\,\text{M}\Omega$, $C = 100\,\text{pF}$, and $L = 1\,\text{H}$, use phasor notation to find the following:

(a) $i(t)$

(b) $V_c(t)$, the voltage across the capacitor

4.13 (i) Show that the locus of the points $P(x, y)$ obeying the equation

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

represents a circle. (ii) Express the coordinates of the center and the radius. Use the following equations of circles to find the centers and radii.

$$x^2 + y^2 + 8x - 4y + 11 = 0$$
$$x^2 + y^2 - 10x - 6y + 9 = 0$$
$$225x^2 + 225y^2 + 90x - 300y + 28 = 0$$

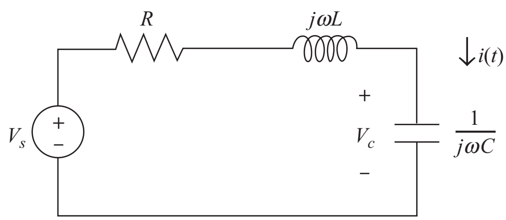

FIGURE MA-6 A series RLC circuit for Problem 4.12.

4.14 Recall the vector identity $\nabla \times \psi\mathbf{A} \equiv \psi\nabla \times \mathbf{A} + \nabla\psi \times \mathbf{A}$, where ψ is a scalar function and $\mathbf{A}$ is a vector point function. Suppose $\mathbf{A} = A_z\mathbf{a}_z$, where $A_z = \dfrac{e^{-jkr}}{r}$ and k is a constant. Simplify $\nabla \times \mathbf{A}$.

4.15 Between two points A and B on the brink of a circular water pond, a transmission line has to be run. It costs twice the money per meter length to install the cable through the water compared to installation on the edge. One might take the cable (a) completely around the arc on the surrounding land or (b) straight through in the water or (c) partly on the arc and for the remaining, straight in the water. (i) What path costs the maximum money? (ii) Suggest an arrangement that minimizes the cost. With some numerical values, plot the cost function.

4.16 Show the following series expansion assuming $|x| < 1$:

$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \cdots$$

PART

1

VECTOR ANALYSIS

CODES OF ETHICS

Engineering is a profession that makes significant contributions to the economic and social well-being of people all over the world. As members of this important profession, engineers are expected to exhibit the highest standards of honesty and integrity. Unfortunately, the engineering curriculum is so crowded that there is no room for a course on ethics in most schools. Although there are over 850 codes of ethics for different professions all over the world, the code of ethics of the Institute of Electrical and Electronics Engineers (IEEE) is presented here to give students a flavor of the importance of ethics in engineering professions.

We, the members of the IEEE, in recognition of the importance of our technologies in affecting the quality of life throughout the world, and in accepting a personal obligation to our profession, its members and the communities we serve, do hereby commit ourselves to the highest ethical and professional conduct and agree:

1. to accept responsibility in making engineering decisions consistent with the safety, health, and welfare of the public, and to disclose promptly factors that might endanger the public or the environment;
2. to avoid real or perceived conflicts of interest whenever possible, and to disclose them to affected parties when they do exist;
3. to be honest and realistic in stating claims or estimates based on available data;
4. to reject bribery in all its forms;
5. to improve the understanding of technology, its appropriate application, and potential consequences;
6. to maintain and improve our technical competence and to undertake technological tasks for others only if qualified by training or experience, or after full disclosure of pertinent limitations;
7. to seek, accept, and offer honest criticism of technical work, to acknowledge and correct errors, and to credit properly the contributions of others;
8. to treat fairly all persons regardless of such factors as race, religion, gender, disability, age, or national origin;
9. to avoid injuring others, their property, reputation, or employment by false or malicious action;
10. to assist colleagues and co-workers in their professional development and to support them in following this code of ethics.

—Courtesy of IEEE

CHAPTER 1

VECTOR ALGEBRA

Books are the quietest and most constant friends; they are the most accessible and wisest of counselors, and most patient of teachers.

—CHARLES W. ELLIOT

1.1 INTRODUCTION

Electromagnetics (EM) may be regarded as the study of the interactions between electric charges at rest and in motion. It entails the analysis, synthesis, physical interpretation, and application of electric and magnetic fields.

Electromagnetics (EM) is a branch of physics or electrical engineering in which electric and magnetic phenomena are studied.

EM principles find applications in various allied disciplines such as microwaves, antennas, electric machines, satellite communications, bioelectromagnetics, plasmas, nuclear research, fiber optics, electromagnetic interference and compatibility, electromechanical energy conversion, radar meteorology, and remote sensing.[1,2] In physical medicine, for example, EM power, in the form either of shortwaves or microwaves, is used to heat deep tissues and to stimulate certain physiological responses in order to relieve certain pathological conditions. EM fields are used in induction heaters for melting, forging, annealing, surface hardening, and soldering operations. Dielectric heating equipment uses shortwaves to join or seal thin sheets of plastic materials. EM energy offers many new and exciting possibilities in agriculture. It is used, for example, to change vegetable taste by reducing acidity.

EM devices include transformers, electric relays, radio/TV, telephones, electric motors, transmission lines, waveguides, antennas, optical fibers, radars, and lasers. The design of these devices requires thorough knowledge of the laws and principles of EM.

[1] For numerous applications of electrostatics, see J. M. Crowley, *Fundamentals of Applied Electrostatics*. New York: John Wiley & Sons, 1986.

[2] For other areas of applications of EM, see, for example, D. Teplitz, ed., *Electromagnetism: Paths to Research*. New York: Plenum Press, 1982.

†1.2 A PREVIEW OF THE BOOK

The subject of electromagnetic phenomena in this book can be summarized in Maxwell's equations:

$$\nabla \cdot \mathbf{D} = \rho_v \tag{1.1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{1.2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{1.3}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \tag{1.4}$$

where ∇ = the vector differential operator
$\mathbf{D}$ = the electric flux density
$\mathbf{B}$ = the magnetic flux density
$\mathbf{E}$ = the electric field intensity
$\mathbf{H}$ = the magnetic field intensity
ρ_v = the volume charge density
$\mathbf{J}$ = the current density

Maxwell based these equations on previously known results, both experimental and theoretical. A quick look at these equations shows that we shall be dealing with vector quantities. It is consequently logical that we spend some time in Part 1 examining the mathematical tools required for this course. The derivation of eqs. (1.1) to (1.4) for time-invariant conditions and the physical significance of the quantities $\mathbf{D}$, $\mathbf{B}$, $\mathbf{E}$, $\mathbf{H}$, $\mathbf{J}$, and ρ_v will be our aim in Parts 2 and 3. In Part 4, we shall reexamine the equations for time-varying situations and apply them in our study of practical EM devices such as transmission lines, waveguides, antennas, fiber optics, and radar systems.

1.3 SCALARS AND VECTORS

Vector analysis is a mathematical tool with which electromagnetic concepts are most conveniently expressed and best comprehended. We must learn its rules and techniques before we can confidently apply it. Since most students taking this course have little exposure to vector analysis, considerable attention is given to it in this and the next two chapters.[3] This chapter introduces the basic concepts of vector algebra in Cartesian coordinates only. The next chapter builds on this and extends to other coordinate systems.

A quantity can be either a scalar or a vector. A scalar is a quantity that is completely specified by its magnitude.

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†Indicates sections that may be skipped, explained briefly, or assigned as homework if the text is covered in one semester.

[3]The reader who feels no need for review of vector algebra can skip to the next chapter.

A **scalar** is a quantity that has only magnitude.

Quantities such as time, mass, distance, temperature, entropy, electric potential, and population are scalars. A vector has not only magnitude, but direction in space.

A **vector** is a quantity that is described by both magnitude and direction.

Vector quantities include velocity, force, momentum, acceleration displacement, and electric field intensity. Another class of physical quantities is called *tensors,* of which scalars and vectors are special cases. For most of the time, we shall be concerned with scalars and vectors.[4]

To distinguish between a scalar and a vector it is customary to represent a vector by a letter with an arrow on top of it, such as $\vec{A}$ and $\vec{B}$, or by a letter in boldface type such as **A** and **B**. A scalar is represented simply by a letter—for example, A, B, U, and V.

EM theory is essentially a study of some particular fields.

A **field** is a function that specifies a particular quantity everywhere in a region.

A field may indicate variation of a quantity throughout space and perhaps with time. If the quantity is scalar (or vector), the field is said to be a scalar (or vector) field. Examples of scalar fields are temperature distribution in a building, sound intensity in a theater, electric potential in a region, and refractive index of a stratified medium. The gravitational force on a body in space and the velocity of raindrops in the atmosphere are examples of vector fields.

1.4 UNIT VECTOR

A vector $\mathbf{A}$ has both magnitude and direction. The *magnitude* of $\mathbf{A}$ is a scalar written as A or $|\mathbf{A}|$. A *unit vector* $\mathbf{a}_A$ along $\mathbf{A}$ is defined as a vector whose magnitude is unity (i.e., 1) and its direction is along $\mathbf{A}$; that is,

$$\boxed{\mathbf{a}_A = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\mathbf{A}}{A}} \tag{1.5}$$

Note that $|\mathbf{a}_A| = 1$. Thus we may write $\mathbf{A}$ as

$$\mathbf{A} = A\mathbf{a}_A \tag{1.6}$$

which completely specifies $\mathbf{A}$ in terms of its magnitude A and its direction $\mathbf{a}_A$.

A vector $\mathbf{A}$ in Cartesian (or rectangular) coordinates may be represented as

$$(A_x, A_y, A_z) \qquad \text{or} \qquad A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z \tag{1.7}$$

[4]For an elementary treatment of tensors, see, for example, A. I. Borisenko and I. E. Tarapor, *Vector and Tensor Analysis with Applications.* New York: Dover, 1979.

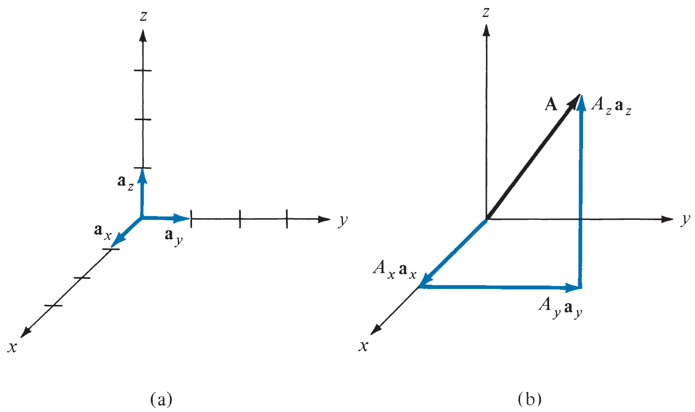

FIGURE 1.1 (**a**) Unit vectors $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$, (**b**) components of **A** along $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$.

where A_x, A_y, and A_z are called the *components of* **A** in the x-, y-, and z-directions, respectively; $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$ are unit vectors in the x-, y-, and z-directions, respectively. For example, $\mathbf{a}_x$ is a dimensionless vector of magnitude one in the direction of the increase of the x-axis. The unit vectors $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$ are illustrated in Figure 1.1(a), and the components of **A** along the coordinate axes are shown in Figure 1.1(b). The magnitude of vector **A** is given by

$$\boxed{A = \sqrt{A_x^2 + A_y^2 + A_z^2}} \tag{1.8}$$

and the unit vector along **A** is given by

$$\mathbf{a}_A = \frac{A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \tag{1.9}$$

1.5 VECTOR ADDITION AND SUBTRACTION

Two vectors **A** and **B** can be added together to give another vector **C**; that is,

$$\mathbf{C} = \mathbf{A} + \mathbf{B} \tag{1.10}$$

The vector addition is carried out component by component. Thus, if $\mathbf{A} = (A_x, A_y, A_z)$ and $\mathbf{B} = (B_x, B_y, B_z)$.

$$\mathbf{C} = (A_x + B_x)\mathbf{a}_x + (A_y + B_y)\mathbf{a}_y + (A_z + B_z)\mathbf{a}_z \tag{1.11}$$

Vector subtraction is similarly carried out as

$$\begin{aligned} \mathbf{D} &= \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) \\ &= (A_x - B_x)\mathbf{a}_x + (A_y - B_y)\mathbf{a}_y + (A_z - B_z)\mathbf{a}_z \end{aligned} \tag{1.12}$$

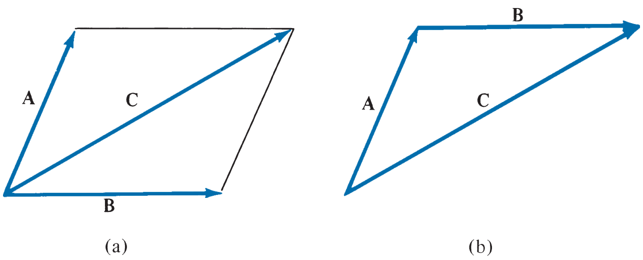

FIGURE 1.2 Vector addition $\mathbf{C} = \mathbf{A} + \mathbf{B}$: (**a**) parallelogram rule, (**b**) head-to-tail rule.

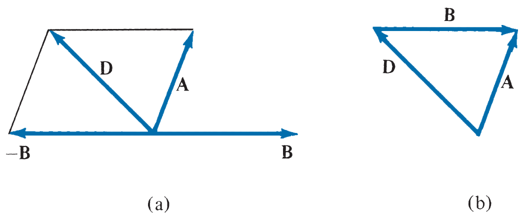

FIGURE 1.3 Vector subtraction $\mathbf{D} = \mathbf{A} - \mathbf{B}$: (**a**) parallelogram rule, (**b**) head-to-tail rule.

Graphically, vector addition and subtraction are obtained by either the parallelogram rule or the head-to-tail rule as portrayed in Figures 1.2 and 1.3, respectively.

The three basic laws of algebra obeyed by any given vectors **A**, **B**, and **C** are summarized as follows:

Law	Addition	Multiplication
Commutative	$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$	$k\mathbf{A} = \mathbf{A}k$
Associative	$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$	$k(\ell\mathbf{A}) = (k\ell)\mathbf{A}$
Distributive	$k(\mathbf{A} + \mathbf{B}) = k\mathbf{A} + k\mathbf{B}$	

where k and ℓ are scalars. Multiplication of a vector with another vector will be discussed in Section 1.7.

1.6 POSITION AND DISTANCE VECTORS

A point P in Cartesian coordinates may be represented by (x, y, z).

The **position vector** $\mathbf{r}_P$ (or **radius vector**) of point P is defined as the directed distance from the origin O to P; that is,

$$\mathbf{r}_P = OP = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z \tag{1.13}$$

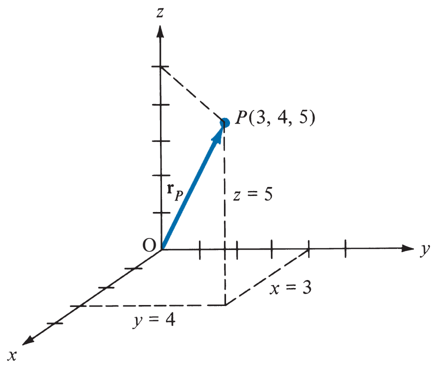

FIGURE 1.4 Illustration of position vector $\mathbf{r}_P = 3\mathbf{a}_x + 4\mathbf{a}_y = 5\mathbf{a}_z$.

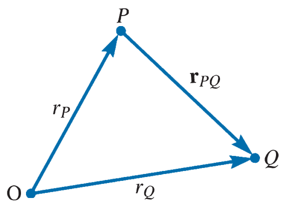

FIGURE 1.5 Distance vector $\mathbf{r}_{PQ}$.

The position vector of point P is useful in defining its position in space. Point (3, 4, 5), for example, and its position vector $3\mathbf{a}_x + 4\mathbf{a}_y + 5\mathbf{a}_z$ are shown in Figure 1.4.

The **distance vector** is the displacement from one point to another.

If two points P and Q are given by (x_P, y_P, z_P) and (x_Q, y_Q, z_Q), the *distance vector* (or *separation vector*) is the displacement from P to Q as shown in Figure 1.5; that is,

$$\begin{aligned} \mathbf{r}_{PQ} &= \mathbf{r}_Q - \mathbf{r}_P \\ &= (x_Q - x_P)\mathbf{a}_x + (y_Q - y_P)\mathbf{a}_y + (z_Q - z_P)\mathbf{a}_z \end{aligned} \tag{1.14}$$

The difference between a point P and a vector $\mathbf{A}$ should be noted. Though both P and $\mathbf{A}$ may be represented in the same manner as (x, y, z) and (A_x, A_y, A_z), respectively, the point P is not a vector; only its position vector $\mathbf{r}_P$ is a vector. Vector $\mathbf{A}$ may depend on point P, however. For example, if $\mathbf{A} = 2xy\mathbf{a}_x + y^2\mathbf{a}_y - xz^2\mathbf{a}_z$ and P is $(2, -1, 4)$, then $\mathbf{A}$ at P would be $-4\mathbf{a}_x + \mathbf{a}_y - 32\mathbf{a}_z$. A vector field is said to be *constant* or *uniform* if it does not depend on space variables x, y, and z. For example, vector $\mathbf{B} = 3\mathbf{a}_x - 2\mathbf{a}_y + 10\mathbf{a}_z$ is a uniform vector while vector $\mathbf{A} = 2xy\mathbf{a}_x + y^2\mathbf{a}_y - xz^2\mathbf{a}_z$ is not uniform because $\mathbf{B}$ is the same everywhere, whereas $\mathbf{A}$ varies from point to point.

EXAMPLE 1.1

If $\mathbf{A} = 10\mathbf{a}_x - 4\mathbf{a}_y + 6\mathbf{a}_z$ and $\mathbf{B} = 2\mathbf{a}_x + \mathbf{a}_y$, find (a) the component of $\mathbf{A}$ along $\mathbf{a}_y$, (b) the magnitude of $3\mathbf{A} - \mathbf{B}$, (c) a unit vector along $\mathbf{A} + 2\mathbf{B}$.

Solution:

(a) The component of $\mathbf{A}$ along $\mathbf{a}_y$ is $A_y = -4$.

(b) $$\begin{aligned} 3\mathbf{A} - \mathbf{B} &= 3(10, -4, 6) - (2, 1, 0) \\ &= (30, -12, 18) - (2, 1, 0) \\ &= (28, -13, 18) \end{aligned}$$

Hence,

$$\begin{aligned} |3\mathbf{A} - \mathbf{B}| &= \sqrt{28^2 + (-13)^2 + (18)^2} = \sqrt{1277} \\ &= 35.74 \end{aligned}$$

(c) Let $\mathbf{C} = \mathbf{A} + 2\mathbf{B} = (10, -4, 6) + (4, 2, 0) = (14, -2, 6)$.

A unit vector along $\mathbf{C}$ is

$$\mathbf{a}_c = \frac{\mathbf{C}}{|\mathbf{C}|} = \frac{(14, -2, 6)}{\sqrt{14^2 + (-2)^2 + 6^2}}$$

or

$$\mathbf{a}_c = 0.9113\mathbf{a}_x - 0.1302\mathbf{a}_y + 0.3906\mathbf{a}_z$$

Note that $|\mathbf{a}_c| = 1$ as expected.

PRACTICE EXERCISE 1.1

Given vectors $\mathbf{A} = \mathbf{a}_x + 3\mathbf{a}_z$ and $\mathbf{B} = 5\mathbf{a}_x + 2\mathbf{a}_y - 6\mathbf{a}_z$, determine

(a) $|\mathbf{A} + \mathbf{B}|$

(b) $5\mathbf{A} - \mathbf{B}$

(c) The component of $\mathbf{A}$ along $\mathbf{a}_y$

(d) A unit vector parallel to $3\mathbf{A} + \mathbf{B}$

Answer: (a) 7, (b) $(0, -2, 21)$, (c) 0, (d) $\pm(0.9117, 0.2279, 0.3419)$.

EXAMPLE 1.2

Points P and Q are located at $(0, 2, 4)$ and $(-3, 1, 5)$. Calculate

(a) The position of vector $\mathbf{r}_P$

(b) The distance vector from P to Q

(c) The distance between P and Q

(d) A vector parallel to PQ with magnitude of 10